MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR DEPARTMENT OF MATHEMATICS CURRICULUM WORKSHOP 3rd and 4th September 2021



M.Sc. (Mathematics)

PROPOSED SYLLABUS September 2021

Malaviya National Institute of Technology Jaipur

Department of Mathematics

Proposed scheme for M.Sc. (Mathematics)

M.Sc. I Semester							
Course Code	Course Name Course L T P						
21MAT	Linear Algebra	4	3	1	0		
21MAT	Abstract Algebra	4	3	1	0		
21MAT	Ordinary Differential Equation	4	3	1	0		
21MAT	Real Analysis	4	3	1	0		
21MAT	Multivariable Calculus	4	3	1	0		
21MAT	Computer Language	1	1	0	0		
21MAP	Computer Lab	2	0	0	4		
	I Semester credits	23					

	M.Sc. II Semester					
Course Code	Course Name	Course credits	L	Т	Р	
21MAT	General Topology	4	3	1	0	
21MAT	Complex Analysis	4	3	1	0	
21MAT	Introduction to Numerical Analysis	4	3	1	0	
21MAT	Partial Differential Equation	4	3	1	0	
21MAT	Probability and Statistics	4	3	1	0	
	Program Elective-I	3	3	0	0	
	II Semester credits	23				

	M.Sc. III Semester					
Course Code	Course Name	Course credits	L	Т	Р	
21MAT	Functional Analysis	4	3	1	0	
	Program Elective-II	3	3	0	0	
	Program Elective-III	3	3	0	0	
	Program Elective-IV	3	3	0	0	
	Program Elective-V	3	3	0	0	
21MAD	Dissertation-I	4	0	0	08	
	III Semester credits	20				

	M.Sc. IV Semester				
Course Code	Course Name	Course credits	L	Т	Р
	Program Elective-VI	3	3	0	0
	Program Elective-VII	3	3	0	0
	Open Elective	3	3	0	0
21MAD	Dissertation-II	8	0	0	16
	IV Semester credits	17			

Type of course	Credits assigned	Range available
Core Course	47	30-50
Program elective	21	15-24
Open elective	3	0-6
Dissertation	12	10-16
Total	83	80-90

Semester	Credits assigned	Range available
Ι	23	20-24
II	23	20-24
III	20	20-24
IV	17	10-18
Total	83	80-90

Total Credits to be earned in order to become eligible for award of M.Sc. (Mathematics) (Two Year Full Time) Degree: 83 Credits

Course Code	: <u>21MAT</u>
Course Name	: Linear Algebra
Credits	: <u>4</u> L- <u>3</u> T- <u>1</u> P- <u>0</u>
Course Type	: Core
Prerequisites	: UG Algebra

Department/Centre : Department of Mathematics

Course Contents

Review of fields, definition and basic examples. Vector spaces, subspaces, linear combinations, spanning sets, basis and dimension, linear transforms. Rank and nullity of linear transformation. Representation of transforms by matrices. Duality and transpose of a linear transformation. Linear functional dual space.

Inner product spaces. Orthogonality, Gram-Schmidt orthogonalization. Orthogonal projections. Primary decomposition theorem (statement only). Linear functional and adjoins. Unitary and normal operators.

Eigenvalues and Eigenvectors, characteristics polynomials, minimal polynomials. Cayley Hamilton's theorem, triangularization, diagonalization. Spectral Theorem for normal operators. Jordan canonical form.

Recommended Readings

Text book:-

- 1. Nair M.T., Singh A., Linear algebra, Springer, 2018.
- 2. Axler S., Linear Algebra Done Right, Springer UTM, 1997.

Reference book:-

- 1. Hoffman K. and Kunze R., Linear Algebra, Prentice Hall of India, 2005
- 2. Vineberg E.B., *A course in algebra*, Graduate text in Mathematics, volume 56, AMS, 2003.
- 3. Rao R.A. and Bhimasankaram P., *Linear Algebra*, Springer, 2000.
- 4. Strang G., Linear Algebra and Its Applications, Pearson, 2005.

Online resources-

1. Linear algebra: NPTEL (https://nptel.ac.in/courses/111/106/111106051/#)

Course Code	: <u>21MAT</u>
Course Name	: Abstract Algebra
Credits	: <u>4</u> L - <u>3</u> T - <u>1</u> P - <u>0</u>
Course Type	: Core
Prerequisites	: Undergraduate Algebra

Course Contents

Review of basic group theory, Normal subgroups, Quotient groups, homomorphism and isomorphism theorems, Maximal subgroups and Composition series of group, Group actions, Cauchy and Sylow's Theorems for finite groups, Internal and External direct product, P-groups, Nilpotent groups, Solvable groups, Structure theorem of finite Abelian groups.

Rings, Integral domain, Properties of ideals, Prime ideals and maximal ideals, Ring homomorphism and Quotient rings, Chinese remainder theorem, Euclidean domain, Euclidean Algorithm, Principal ideal domain, Unique factorization domain, Polynomial rings.

Field extensions, Basic properties of degree of field extensions.

Department/Centre : Department of Mathematics

Recommended Readings

Text Books: -

- 1. Dummit D. S., Foote R. M., Abstract Algebra, 3rd Edition, John Wiley & Sons, NY, 2011.
- 2. Artin M., *Algebra*, 2nd Edition, Pearson Education, 2011.

Reference books: -

- 1. Herstein I. N., *Topics in Algebra*, 3rd Edition, John Wiley & Sons, 1996.
- 2. Fraleigh J., A First Course in Abstract Algebra, 8th Edition, Pearson Education, 2020.
- 3. Gallion J. A., *Contemporary Abstract Algebra*, 9th Edition, Brooks/Cole Cengage Learning, 2017.
- 4. Hungerford T.A., Algebra, Graduate Texts in Mathematics, Springer-Verlag, 1980.
- 5. Lang S., Algebra, 3rd Edition, Addison-Wesley, 1999

Online/E resources: -

https://nptel.ac.in/courses/111/106/111106113/ https://nptel.ac.in/courses/111/106/111106131/

Course Code	:	21M	AT			
Course Name	:	Ordi	nary Differenti	al Equation		
Credits	:	4	L - 3	T - 1	P - 0	
Course Type	:	Core				
Prerequisites	:		vledge of linear icients	differential equa	ations with constan	t/variable

Department/Centre : Department of Mathematics

Course Contents

Series solution. Real analytic solutions, Taylor's Series, Ordinary and singular points. Radius of convergence .Series solution near an ordinary point. Series solution near a regular singular point (Method of Frobenius).

Existence and Uniqueness of Initial Value Problems: Picard's and Peano's Theorems. Boundary Value Problems for Second Order Equations.

Orthogonal functions and orthonormal sets, Oscillations of second order equations, Sturm comparison theorems, Elementary linear oscillations, Eigenvalues and Eigenfunctions. Sturm-Liouville system. Green's functions.

System of simultaneous linear differential equations with constant and variable coefficients. Asymptotic behavior and Stability theory.

Recommended Readings

Text Books:-

- 1. Ahmad S. and Ambrosetti A., A Text Book on Ordinary Differential Equations, Springer, 2015.
- 2. Nandakumaran A.K., Datti P.S. and George R.K., *Ordinary Differential Equations: Principles and Applications*, Cambridge University Press, 2017.
- 3. Simmons G.F. and Krantz S.G., *Differential Equations Theory, Techniques and Practice*, McGraw Hill, 2006.

Reference books:-

- 1. Deo S. G, Raghvendra V., Kar R. and Lakshmikantham V., *Textbook of Ordinary Differential Equations*, McGraw Hill Education, 2018.
- 2. Braun M., Differential Equations and Their Applications, Springer, 1993.
- 3. Coddington E.A. and Levison N., *Theory of Ordinary Differential Equations*, McGraw Hill, 2017.

Department/Centre : Department of Mathematics						
Course Code	:	21M	АT			
Course Name	:	Real	Analysis			
Credits	:	4	L - <u>3</u>	T - <u>1</u>	P 0	
Course Type	:	Core				
Prerequisites	:	Unde	rgraduate Calcu	ılus		

Course Contents

Review of the real number system, definition of a metric space and its examples, convergence of sequences in metric spaces, Subsequences, Cauchy sequences and complete metric spaces, Cantor Intersection Theorem, Baire category theorem.

Limits and Continuity of a function defined on a metric space, uniform continuity, completion of a metric space, contraction mapping, Banach fixed point theorem, connectedness, compactness, Heine Borel theorem.

Monotonic functions, Functions of bounded variation, Partition, Riemann-Stieltjes sum, Riemann-Stieltjes integral, properties, step function as integrator, reduction of RS integral to a finite sum, upper and lower Stieltjes sums, upper and lower integrals, Riemann condition, comparison theorem, mean value theorems, second mean value theorem for RS integrals.

Sequences and series of functions, Pointwise convergence, uniform convergence and its relation to continuity, differentiation and integration of series of functions, Cauchy condition for uniform convergence of series, Power Series, Weierstrass approximation theorem (only statement), equicontinuity, Arzela-Ascoli theorem and applications.

Recommended Readings

Text books-

- 1. Carothers N.L., Real Analysis, Cambridge University Press, Indian Edition, 2009.
- 2. Shirali S. and Vasudeva H.L., *Metric Spaces*, Springer, South Asian edition, 2019.

Reference books-

- 1. Ghorpade S.R. and Limaye B.V., A Course in Calculus and Real Analysis, Springer, 2006.
- 2. Royden H.L., Real Analysis, McMillan Publication Co. Inc. New York, 1988.
- 3. Apostol T.M., Mathematical Analysis, Narosa Publishing House, 1985.
- 4. Rudin W., Principles of Mathematical Analysis, McGraw Hill, Singapore, 1976.

Online resources-

1. <u>https://nptel.ac.in/courses/111/106/111106053/</u> (By Prof. S. H. Kulkarni, IIT Madras)

Course Code	: 21MAT	
Course Name	: Multivariable Calculus	
Credits	: <u>4</u> L - <u>3</u> T - <u>1</u> P - <u>0</u>	
Course Type	: Core	
Prerequisites	: Undergraduate Calculus	

Department/Centre : Department of Mathematics

Course Contents

Limits and Continuity of functions defined on Euclidean Spaces: Review of vector algebra in \mathbb{R}^n . Real-valued functions of several variables. Level sets (level curves, level surfaces, etc.). Vector valued functions of several variables. Sequences in \mathbb{R}^n and their limits. Neighborhoods in \mathbb{R}^n . Limits and continuity of scalar- and vector-valued functions of several variables.

Differentiation: Partial derivatives. Differentiability of a real-valued function of several variables, the concept of (total) derivative. Gradient and directional derivatives. Chain Rule. Euler's Theorem. Higher order partial derivatives. Mixed Derivative Theorem. Mean Value Theorem and Taylor's Theorem for functions of several variables. Review of quadratic forms. Hessian matrix. Local maxima/minima and saddle points. Constrained maxima and minima of real-valued functions of several variables. Differentiation of vector-valued functions of several variables. Differentiation in R^n . Implicit function theorem, Inverse function theorem.

Multiple Integrals: Definition of double (resp: triple) integral of a function defined and bounded on a rectangle (resp: box). Geometric interpretation. Basic properties of double and triple integrals. Iterated integrals, Fubini's Theorem. Integrability and the integral over arbitrary bounded domains. Change of variables formula (Statement only). Polar, cylindrical and spherical coordinates, and integration using these coordinates.

Line Integrals: Paths (parameterized curves) in \mathbb{R}^n , Smooth and piecewise smooth paths. Closed paths. Equivalence and orientation preserving equivalence of paths. Definition of the line integral of a vector field over a piecewise smooth path. Basic properties of line integrals. First and Second Fundamental Theorems of Calculus for Line Integrals. Green's Theorem (proof only in the case of rectangular domains) and its applications to evaluation of line integrals.

Surface Integrals: Parameterized surfaces. Smoothly equivalent parameterizations. Area of such surfaces. Definition of surface integral. Curl and divergence of a vector field. Stokes' Theorem (proof assuming the general form of Green's Theorem), Gauss' Divergence Theorem (proof only in the case of cubical domains) and their applications.

<u>Recommended Readings</u> Text books:

- 1. Apostol T., Calculus, Vol. 2, Second Ed., John Wiley, New York, 2002.
- 2. Ghorpade S.R. and Limaye, B.V., A Course in Multivariable Calculus and Analysis, Springer 2010.

Reference books:

- 1. Edwards C.H., Advanced calculus of several variables, Dover Publications Inc., 1995.
- 2. Apostol T., Mathematical Analysis, Second Ed., Narosa, New Delhi, 1974.
- 3. Courant R. and John F., *Introduction to Calculus and Analysis*, Vol. 2, Springer-Verlag, New York, 1989.
- 4. Fleming W., *Functions of Several Variables*, Second Ed., Springer-Verlag, New York, 1977.
- 5. Shifrin T., *Multivariable Mathematics: Linear Algebra, Multivariable Calculus, and Manifolds,* Wiley India Pvt. Ltd., 2018.
- 6. Marsden J.E. and Tromba A.J., *Vector Calculus*, Fourth Ed., W. H. Freeman and Co., New York, 1996.
- 7. Widder D.V., Advanced Calculus, Second Ed., Dover Pub., New York, 1989.
- 8. Shirali S. and Vasudeva H.L., Multivariable analysis, Springer, New York, 2011.

Online resources-

1. NPTEL course: *Multivariable Calculus* coordinated by IIT Roorkee available at the link: <u>https://nptel.ac.in/courses/111/107/111107108/</u> (as on 3rd August 2021).

Department/Centre : Department of Mathematics

Course Code	: _21MAT
Course Name	: Computer Language
Credits	: <u>1</u> L- <u>1</u> T- <u>0</u> P- <u>0</u>
Course Type	: Core
Prerequisites	: None

Course Contents

Introduction to C, Algorithms and Flowcharts, Constants, Variables, Data Types, Declarations, Operators and Expressions, Operator Precedence and Associativity, Input and Output Operations, Formatting, Decision Making, Branching, Looping, Arrays, Character Arrays and Strings

Recommended Readings

Text book-

1. Balagurusamy E., Programming in ANSI C, McGraw Hill, 8th Edition, 2019.

Reference book-

1. Kernighan B. W. and Ritchie D. M., *The C Programming Language (Ansi C Version)*, PHI Learning, 2015.

Online resources-

1. NPTEL course: Introduction to Programming in C By Dr. Satyadev Nandakumar, IIT Kanpur (<u>https://onlinecourses.nptel.ac.in/noc19_cs42/preview</u>)

Course Code	: <u>21MAP</u>
Course Name	: Computer Lab
Credits	: <u>2</u> L- <u>0</u> T- <u>0</u> P- <u>4</u>
Course Type	: Core
Prerequisites	: None

Course Contents

Programming Assignments: Computer Language based theory

Department/Centre : Department of Mathematics

Programming Assignment Manual will be prepared and will be the reference for the questions, assignments, evaluation and the laboratory practices.

Course Code	: <u>21MAT</u>	
Course Name	: General Topology	
Credits	: <u>4</u> L- <u>3</u>	T - <u>1</u> P - <u>0</u>
Course Type	: Core	
Prerequisites	: Real Analysis	

Department/Centre : Department of Mathematics

Course Contents

Definition of a topological space, open and closed sets, metric space as an example of a topological space, neighbourhoods, bases and subbases, limit points, closures, interiors, convergence of sequences in topological space, insufficiency of sequences and first countability, subspace topology, hereditary properties.

Continuous maps, open and closed maps, homeomorphisms, separable spaces, second countability, quotient topology, weak topology, product topology, metrizable spaces, completely metrizable spaces and function spaces.

T₁-axiom, Hausdorff spaces, regularity, complete regularity, normality, Urysohn Lemma, Tychonoff embedding and Urysohn Metrization Theorem, Tietze's Extension Theorem.

Directed sets, Nets, Connectedness, path connectedness, component and path components, locally connected spaces, Lindelöf spaces, compact spaces and various types of compactness: countably compact, sequentially compact, pseudocompact spaces and their relationships, Tychonoff Theorem, One point Compactification.

Recommended Readings

Text book-

- 1. Patty C.W., Foundation in General Topology, Jones and Bartlett, 2010.
- 2. Munkres J. R., *Topology*, 2nd Edition, Pearson Education (India), 2001.
- 3. Armstrong M.A., Basic Topology, Springer (India), 2004.

Reference book-

- 1. Joshi K.D., Introduction to General Topology, New Age International, 2000.
- 2. Simmons G.F., Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
- 3. Kelley J.L., General Topology, Springer, Van Nostrand, 1955.
- 4. Willard S., *General Topology*, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1970.

Online resources-

1. https://nptel.ac.in/courses/111/106/111106054/

Department/Centre : Department of Mathematics									
Course Code	:	21MA7	ſ						
Course Name	:	Comple	Complex Analysis						
Credits	:	4	L - 3	T - 1	P - 0				
Course Type	:	Core							
Prerequisites	:	None							

Department/Centre : Department of Mathematics

Course Contents

Topology of Complex number, Stereographic projection, Function of complex variable, Limits, Continuity, Uniform Continuity and Differentiability of function of complex variable, Exponential function, Trigonometric and inverse trigonometric functions, Logarithm function, Cauchy Riemann equation, Harmonic function, Harmonic conjugate.

Analytic function, Construction of analytic function, Entire function, Bilinear transformation, Problems on cross ratio, Fixed point, Normal form of bilinear transformation.

Zeros and singularities of analytic function, types of singularities: isolated or non-isolated singularity, classification of singularities: removable singularity, poles, essential and non-isolated essential, Power series, circle of convergence, radius of convergence, Abel's formula, Cauchy Hadmard formula.

Curve, Jordan curve, Smooth curve, Winding number, Complex integration, Cauchy integral theorem, Cauchy Goursat theorem, Cauchy's integral formulas, ML inequality, derivative of analytic function, Morera's theorem, Lucca theorem.

Liouville's theorem, Fundamental theorem of algebra, Identity theorem, Extension of Liouville's theorem, Taylor's Expansion, Laurent's series, Residue at a pole and at infinity, Cauchy's Residue theorem, Residue and Contour integration.

Meromorphic function, Rational function, Picard's little theorem, Argument principle, Maximum modulus and minimum modulus principle, Rouche's theorem, Conformal Mapping, Fundamental transformation, Schwarz lemma.

Recommended Readings

Text book-

- 1. Ponnusamy S., Silverman H., Complex Variable with Applications, Birkhauser, 2006.
- 2. Kasana H.S., *Complex Variables: Theory and Application*, PHI Intl Ltd, 2nd Edition, 2017.

Reference book-

- 1. Churchill R.V., Brown J.W., *Complex Variable & Applications*, Tata McGraw Hill Education, 2009.
- 2. Conway J.B, Function of One Variable, Narosa Distributors Pvt. Ltd., 1973.
- 3. Ahlfors L.V., *Complex Analysis*, McGraw Hill, 3rd Edition, 1979.
- 4. Zill D.G., Patrick D.S., A First Course in Complex Analysis with Application, Jones & Bartlett publications, 2013.
- 5. Daniel A., A Complex Analysis Problem Book, Birkhauser, 2010.

Department/Centre : **Department of Mathematics**

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Course Code	: <u>21MAT</u>
Course Name	: Introduction to Numerical Analysis
Credits	: <u>4</u> L- <u>3</u> T- <u>1</u> P- <u>0</u>
Course Type	: Core
Prerequisites	: Basic linear algebra and ordinary differential equations

Course Contents

Numerical Computations and Errors: Preliminaries of computing. Floating-point approximation of a number, Loss of significance and error propagation, Stability in numerical computation. Norms of vectors and matrices. Review of Taylor's series.

Linear Systems: Gaussian elimination with pivoting strategies, Gauss Jordan elimination. Iterative methods (Jacobi and Gauss-Seidal method) with convergence analysis.

Nonlinear Equations: Bisection method, Regula-falsi method, Secant method, Fixed-point iteration, Newton-Raphson method. Rate of convergence through error analysis. Solution of a system of nonlinear equations by fixed-point iteration and Newton's method.

Interpolation: Interpolation and Error. Finite difference operators. Newton's forward and backward difference interpolation formulae, Central difference formulae. Lagrange's interpolation, Newton's divided difference formulae for unequal interval. Spline interpolation.

Numerical Differentiation and Integration: Numerical differentiation, error estimates, Maxima/minima of tabulated functions. Newton-Cotes formulae with associated errors. Gaussian quadrature. Richardson extrapolation.

Numerical Solution of ODEs: Taylor's series method, Picard's method, Euler's and modified Euler's methods, Runge-Kutta methods. Solution of simultaneous and higher order equations. Predictor-Corrector methods. Single step and Multi-step methods: order, consistency, stability and convergence analysis. Two-point BVPs: Shooting and Finite difference method.

<u>Recommended Readings</u> Text books:

- 1. Atkinson K.E., An Introduction to Numerical Analysis, 2nd Edition, Wiley-India, 1989.
- 2. Burden R.L. and Faires J.D., Numerical Analysis, 9th Edition, Cengage learning, 2011.

Reference books:

- 1. Jain M.K., Iyengar S.R.K. and Jain R.K., *Numerical Methods for Scientific and Engineering Computation*, Wiley Eastern Limited, 2012.
- 2. Conte S.D. and Boor C., *Elementary Numerical Analysis An Algorithmic Approach*, 3rd Edition, McGraw-Hill, 1981.
- 3. Gerald C.F. and Wheatly P.O., *Applied numerical analysis*, Seventh Edition, Pearson Addison-Wesley Pub. Co, 1985.

- 4. Sastry S. S., Introductory Methods of Numerical Analysis, Prentice Hall of India, 2012.
- 5. Sharma J.N., *Numerical methods for Engineers and Scientists*, 2nd edition, Narosa Publishing House New Delhi, 2008.

Online resources-

1. NPTEL course: *Numerical Analysis* coordinated by IIT Madras available at the link: <u>https://nptel.ac.in/courses/111/106/111106101/</u> (as on 3rd August 2021.)

Department	Department of Mathematics					
Course Code	: 21MAT					
Course Name	: Partial Differential Equations					
Credits	: <u>4</u> L- <u>3</u> T- <u>1</u> P- <u>0</u>					
Course Type	: Core					
Prerequisites	Basic Multivariable Calculus and ODEs					

COURSE CONTENTS

Classification of first order PDE, Method of characteristics, Cauchy problem, Cauchy-Kowalewsky theorem, Holmgren's Uniqueness Theorem, Integral surfaces passing through a given curve, Nonlinear first-order PDEs, Compatible systems, Classification and Canonical forms of second order PDE, Charpit's method, Jacobi's method for nonlinear PDEs.

Laplace equation: Fundamental solution, Construction of Green's function for Dirichlet problem posed on special domains. Poisson's formula, Solution of Dirichlet problem on a rectangle by method of separation of variables, Mean value property, Maximum principles, Dirichlet principle

Heat equation: Fundamental solution, Solution of initial-boundary value problem by separation of variables method, Maximum principle.

Wave equation: D'Alembert's formula, Solution of wave equation on bounded domains by separation of variables method, Duhamel's principle, Solutions by spherical means, Non-Homogeneous Problems, Duhamel's principle, Energy Methods.

Recommended Readings

Text Books:

- 1. Evans L.C., Partial Differential Equations, AMS (1998).
- 2. McOwen R.C., *Partial Differential Equations Method and Applications*, Pearson Education, second Edition (2005).

Reference books:

- 1. Myint-U T. and Debnath L., *Linear Partial Differential Equations for Scientists and Engineers*, Birkhauser, Boston, 2007.
- 2. Nandakumaran A.K. and Datti P.S., *Partial Differential Equations-Classical Theory with a Modern Touch*, Cambridge (2020) (Cambridge-IISc Series).
- 3. Pinchover Y. and Rubinstein J., An Introduction to Partial Differential Equations, Cambridge, 2005.
- 4. Fritz J., *Partial Differential Equations*, Applied Mathematical Sciences, 1. Springer-Verlag, New York, 1982.
- 5. Farlow S.J., Partial Differential Equations for Scientists and Engineers, Birkhauser, New York, 1993.

Department/Centre : Department of Mathematics								
Course Code	:	21M	AT					
Course Name	:	Prot	ability a	and Sta	atistics			
Credits	:	4	L -	3	Т-	1	P - 0	
Course Type	:	Core						
Prerequisites	:	none;	[preferr	ed – ur	nderstandi	ng of t	basic statistics]	

Course Contents

Random Experiment, Sample Space, Events – Simple, Composite, Mutually Exclusive and Exhaustive Events, algebra of events. Various Definitions of Probability, Properties of probability function, Addition Theorem, Boole's and Bonferroni's Inequalities, Conditional Probability, Multiplication Theorem, Independence of Events, Total probability, Baye's Theorem.

Random variables, Discrete and Continuous random variables and their expected value, mean, Variance and moment generating function. Specific discrete and continuous distributions, e.g. Binomial, Poisson, Geometric, Pascal, Hypergeometric, Uniform, Exponential, Weibull, Beta, Gamma, Erlang and Normal distributions.

Sampling from a distribution: random sample, concept of derived distribution of functions of random variables, concept of a statistic and its sampling distribution. Point and interval estimation, concept of bias and standard error of an estimate, standard error of sample mean, sample proportion and sample variance. Sampling distribution: χ -square -distribution, t-distribution and F-distribution and their properties. Test of significance: null and alternative hypothesis, errors in sampling (Type I and Type II), critical region, level of significance, Z-test, t-test, χ - test, F-test. χ -Square test as a test of goodness of fit.

Recommended Reading

Text books-

- 1. Ravichandran J., Probability and Statistics for Engineers, Wiley India, 2019.
- 2. Shanker R.G., *Probability and Statistics for Science and Engineering*, Universities Press, 2011.

Reference books-

- 1. Feller W., *An Introduction to Probability Theory and its Applications*, Vol. I and II. New York, NY: Wiley, 1968-1971.
- 2. Jonson R.A , Miller and Freund's *Probability and Statistics for Engineers*, PHI, India Learning Private Limited; 8th edition (1 January 2011).
- 3. Ross S.M., A first course in Probability, Pearson, 2006.

- 4. Gupta S.C. and Kapoor V.K., *Fundamentals of Mathematical Statistics*, Sultan Chand & Sons (1 January 2014).
- 5. Freund W.J., *Mathematical Statistics*, 5th Ed., Prentice-Hall, Inc., Englewood Cliffs, N.J., 1994.
- 6. Hogg R.V., McKean J.W., and Allen T. Craig: *Introduction to Mathematical Statistics*, 7th Edition, Pearson Education, Asia, 2014.
- 7. Ross S.M., Introductory Statistics, Elsevier, 2010.

Department/Centre : Department of Mathematics

Course Code	: <u>21MAT</u>
Course Name	: Functional Analysis
Credits	: <u>4</u> L - <u>3</u> T - <u>1</u> P - <u>0</u>
Course Type	: Core
Prerequisites	: Real analysis, Linear algebra

Course Contents

Normed linear spaces and Banach spaces, Finite dimensional normed spaces, Riesz Lemma, compactness of closed unit disk, equivalence of norms, Schauder basis and separablilty. Bounded linear operators, continuous linear functionals, normed spaces of bounded linear operators, dual space, reflexive space and canonical mapping, weak and weak^{*} topologies.

Hahn-Banach extension theorem, Uniform boundedness principle, open mapping theorem, closed graph theorem and their applications.

Inner product and Hilbert Spaces, orthogonal complements, Orthonormal sets, Total orthonormal sets, Bessel's Inequality, Parseval's relation, Riesz representation theorem. Adjoint operators in Hilbert Spaces, sesquilinear form, Compact Operators and Spectral Theorem, normal operator, unitary operators, Projections and orthogonal projections in Hilbert spaces, Best approximations in Hilbert Spaces.

Recommended Readings

Text books-

- 1. Nair M.T., Functional Analysis: A first course, PHI, 2002.
- 2. Kreyszig E., *Introductory functional analysis with applications*, Wiley publications, 1989.

Reference books-

- 1. Limaye, B.V., Functional Analysis, Wiley Publication, 1996.
- 2. Brown, A.L. and Page A., Functional Analysis, Van Nostrand Reinhold, 1970.
- 3. Kesavan S., Functional Analysis, Hindustan Book Agency, 2014.

Online resources-

- 1. NPTEL Course- https://nptel.ac.in/courses/111/105/111105037/
- 2. MITCourse- https://ocw.mit.edu/courses/mathematics/18-102-introduction-to-functionalanalysis-spring-2009/

M. Sc. Electives II to IV Semester

S.No.	Course	Course Name	L	Т	P	C
	Code					
1.	21MAT	Special Functions	3	0	-	3
2.	21MAT	Applied Linear Algebra	3	0	-	3
3.	21MAT	Commutative Algebra	3	0	-	3
4.	21MAT	Field and Galois Theory	3	0	-	3
5.	21MAT	Graphs and Matrices	3	0	-	3
6.	21MAT	Advanced Functional Analysis	3	0	-	3
7.	21MAT	Probability Theory	3	0	-	3
8.	21MAT	Flow and Thermal Instability	3	0	-	3
9.	21MAT	Fluid Dynamics	3	0	-	3
10.	21MAT	Measure and Integration	3	0	-	3
11.	21MAT	Advanced Analysis	3	0	-	3
12.	21MAT	Introduction to Operator Theory	3	0	-	3
13.	21MAT	Geometric Function Theory	3	0	-	3
14.	21MAT	Fractional Calculus and its Application	3	0	-	3
15.	21MAT	Calculus of variations and tensor analysis	3	0	-	3
16.	21MAT	Fractional differential equations	3	0	-	3
17.	21MAT	Dynamical Systems	3	0	-	3
18.	21MAT	Numerical Linear Algebra	3	0	-	3
19.	21MAT	Finite Element Method	3	0	-	3
20.	21MAT	Integral Equations	3	0	-	3

Open Elective

S.No.	Course	Course Name	L	Τ	P	C
	Code					
1.	21MAT	Number theory	3	0	-	3
2.	21MAT	Discrete Mathematics	3	0	-	3
3.	21MAT	Classical Mechanics	3	0	-	3
4.	21MAT	Introduction to Graph theory	3	0	-	3
5.	21MAT	Integral Transforms	3	0	-	3

Course Code	: <u>21MAT</u>
Course Name	: Special Functions
Credits	: 3 L-3 T-0 P-0
Course Type	: Elective
Prerequisites	: Ordinary Differential equations, Differential and Integral calculus

Department/Centre : Department of Mathematics

Course Contents

The Gamma and Beta Function: Preliminaries, Euler's integral for Gamma (Z), Gamma and Beta functions and its elementary properties Factorial function, Gauss Multiplication formula, Legendre's duplication formula, Gauss multiplication theorem, Incomplete gamma function, Incomplete beta function, Mittag-leffler's function, Riemann Zeta Function. **The Hypergeometric Function :** Definition, Integral representation of hypergeometric function, Transformations, Gauss's hypergeometric functions and its elementary properties, Gauss's hypergeometric differential equation and its solution, Gauss's hypergeometric function, Evaluation of hypergeometric function, relations of contiguity, theorem due to Kummer's, Generalized Hypergeometric series, the function of $_{u}F_{v}$, Bilateral hypergeometric series.

Kummer's function (The Confluent hypergeometric function): Definitions and some elementary results, Recurrence relations, The differential equation, Kummer's first and second formula, Addition and multiplication theorems, Integral representations, Basic properties of ₁F₁, Special cases and its relation to other functions, Products of Kummer's functions.

Bessel functions: Bessel differential equation and its solution, Bessel's functions $J_u(x)$, recurrence relation, generating functions, integral representation, and orthogonality of Bessel functions, modified Bessel function and its properties.

Legendre functions : Legendre's differential equation and its solution, Relations between Legendre functions, The function P(x) and Q(x), Multiplications of two Legendre functions, Integral representations, Integrals involving Legendre functions, Associated Legendre functions.

Recommended Readings

Text book-

- 1. Rainville E.D., Special Function, The Macmillan Company, New York, 1960.
- 2. Sharma J.N. and Gupta R.K., Special Functions, Krishna's Educational Publishers..

Reference book-

- 1. Bell W.W., *Special Function for Scientists and Engineers*, D. Van Nostrand Company Ltd., 1968.
- 2. Andrews G.E., Askey R., Roy R., Special Function, Cambridge University Press, 1999.
- 3. Wang Z.X., Guo D.R., Special Function, World Scientific, 2010.
- 4. Prudnikov A.P., Brychkov Y.A., Marichev O.I., Gould G.G., *Integral and Series*, Gordon and Breach Science Publ., 1990.
- 5. Magnus W., Oberhettinger F., Soni R.P., Formulas and Theorems for the Special Functions of Mathematical Physics, 3rd Edition, Springer-Verlag Berlin Heidelberg GmbH, 1966.
- 6. Larry C.A, *Special Function of Mathematics for Engineers*, 2nd Edition, Oxford University Press, 1998.

Online resources-

Department/Ce	tre : Department of Mathematics	
Course Code	: <u>21MAT</u>	
Course Name	: Applied Linear Algebra	
Credits	: <u>3</u> L- <u>3</u> T- <u>0</u> P- <u>0</u>	
Course Type	: Elective	
Prerequisites	: Linear Algebra	

Course Contents

Review on Inner Product Spaces, projection, orthogonal projection, best approximation, Finite Dimensional spectral theory for normal operators and self adjoint operators. Quadratic forms, Bilinear forms.

Orthogonal reduction, Discrete Fourier Transformation, Complementarity Subspaces, rangenull space decomposition, orthogonal decomposition, Singular-value decomposition, Polar decomposition, Matrix Norms and location of eigenvalues, generalized inverse of a matrix. Perron Frobenius Theory: Positive matrices, Nonnegative matrices, stochastic matrices and applications to Markov chains.

Recommended Readings

Text book-

- 1. Hoffman K., Kunze R., *Linear Algebra*, Prentice-Hall, Second Edition, 2008.
- 2. Meyer C.D., Matrix Analysis and Applied Linear Algebra, SIAM, 2001.
- 3. Horn R., Johnson C. R., Matrix Analysis, Cambridge University Press, New York, 1985.

Reference book-

- 1. Lay D.C., Lay S.R., McDonald J.J., Linear Algebra and Its Applications, Pearson, 2015.
- 2. Zhang F., Matrix Theory-Basic Results and Techniques, Second Edition, Springer, 2011.
- 3. Golub G.H., Van Loan C.F., *Matrix Computations*, Fourth Edition, Hindustan Book Agency, 2013.

Department/Ce	entre	e : <u>D</u>	Department of 1	Mathematics		
Course Code	:	21M	AT			
Course Name	:	Com	mutative Algeb	ora		
Credits	:	3	L - 3	T - 0	P 0	
Course Type	:	Electi	ve			
Prerequisites	:	Abstr	act Algebra			

Course Contents

Revision of Rings and Ideals: Prime and maximal ideals. Chinese remainder theorem, Nilradical, Jacobson radical, operations on ideals, extension and contraction.

Basics of Modules: Module, submodule, quotient module, sums and products, Nakayama's lemma; Homomorphism, kernel, cokernel, direct sum, direct product, universal properties, free module, exact sequences, tensor product of modules and its exactness property, Localization of rings and modules.

Chain conditions: Noetherian rings, Hilbert basis theorem, Primary decomposition of ideals in Noetherian rings. Artinian rings and modules.

Integral Extensions: Integral dependence, going-up and going down theorems, Noether's normalization lemma, Hilbert Nullstellensatz, Krull's principal ideal theorem and dimension theorem.

Recommended Readings

Text book

1. Atiyah M.F. and MacDonald I. G., Introduction to commutative algebra, CRC Press, 2018.

Reference book

- 1. Matsumura H., *Commutative ring theory*, Cambridge Studies in Advanced Mathematics No. 8, Cambridge University Press, 1989.
- 2. Gopalakrishnan N.S., Commutative Algebra, Oxonian Press, 2015.

Online resources-

NPTEL Course: https://nptel.ac.in/courses/111/106/111106098/

Department/Centre : Department of Mathematics								
Course Code	:	21MAT	21MAT					
Course Name	:	Field a	nd Galois Th	leory				
Credits	:	3	L - <u>3</u>	T - 0	P - 0			
Course Type	:	Electiv	e					
Prerequisites	:	Abstrac	t Algebra					

Course Contents

Field Theory: Review of basics of rings, integral domains and fields, Characteristic and prime subfields. Field extensions, Finite, algebraic, and finitely generated field extensions, Classical ruler and compass constructions, Splitting fields and normal extensions, algebraic closures. Finite fields. Quadratic and cyclotomic fields. Separable and inseparable extensions. Norm, trace and discriminant.

Galois Theory: Galois extensions, Galois groups, Fundamental Theorem of Galois Theory, Composite extensions, Examples (including cyclotomic extensions and extensions of finite fields). Solvability by radicals, Galois' Theorem on solvability.

Recommended Readings

Text book-

- 1. Artin M., Algebra, Prentice Hall of India, 1994.
- 2. Dummit D. S., Foote R. M., Abstract Algebra, 3rd Ed., John Wiley, 2004.

Reference book-

- 1. Gallian J.A., *Contemporary Abstract Algebra*, 9th Ed., Brooks/ Cole Cengage Learning, 2017.
- 2. Lang S., *Algebra*, 3rd Ed., Springer, New York, 2002.
- 3. Hungerford T.W., *Algebra*, Springer, 2003.

Department/Centre : Department of Mathematics				
Course Code	: <u>21MAT</u>			
Course Name	: Graphs and Matrices			
Credits	: <u>3</u> L- <u>3</u> T- <u>0</u> P- <u>0</u>			
Course Type	: Core / Elective			
Prerequisites	: Linear Algebra and Basic Graph Theory			

Course Contents

Matrices, Eigenvalues of symmetric matrices, Graphs, Incidence matrix, Rank, Minors, Matchings in bipartite graphs. Adjacency Matrix, Eigenvalues of some graphs, Determinant, Bounds, Energy of a graph, Anti Adjacency matrix of a directed graph, Non-singular trees. Laplacian matrix, Basic properties, Computing Laplacian Eigenvalues, Matrix-tree theorem, Bounds of Laplacian spectral radius, Edge-Laplacian of a tree. Cycles and cuts, Fundamental cycles and Fundamental cuts, Fundamental matrices, Minors. Regular graphs, strongly regular graphs and friendship theorem, Graph with maximum Energy. Line Graph of tree, Block graphs, Signless Laplacian matrix, Nullity of the line graph of a tree. Algebraic connectivity, classification of trees, Monotonicity properties of Fielder vector, Bounds of algebraic connectivity. Distance matrix of a graph, Distance and Laplacian matrix of a tree. Chordal graphs, positive definite completion.

Recommended Readings

Text book-

1. Bapat R., *Graphs and Matrices*, Text and Readings in Mathematics, Second Edition, Hindustan Book Agency, New Delhi, 2014.

Reference book

- 1. Cvetkovic D., Rowlinson., Simic S., *An introduction to the theory of Graph Spectra*, First Edition, Cambridge University press, 2010.
- 2. Balakrishnan R., Ranganathan K., *A textbook of Graph Theory*, Second Edition, Springer, 2012.
- 3. Biggs N., Algebraic Graph Theory, Second Edition, Cambridge University press, 1993.

Department/Centre : Department of Mathematics					
Course Code	21MAT				
Course Name	: Advanced Functional Analysis				
Credits	: <u>3</u> L- <u>3</u> T- <u>0</u> P- <u>0</u>				
Course Type	: Elective				
Prerequisites	: Functional Analysis				

Course Contents

Topological vector spaces (TVS): Definition of a TVS, Separation properties, balanced, convex and absorbing sets in TVS and their properties, special neighborhood bases, quotient and product spaces, finite dimensional TVS, linear maps.

Locally Convex Spaces (LCS): Definition of a locally convex space, special neighborhood bases, seminorms, Minkowski functional, relation between seminorms and locally convex spaces, metrizable LCS, bounded sets and their properties, normability of LCS, Hahn Banach theorem, Dual of a locally convex space, polars and their basic properties.

Topologies associated with polars: weak and weak*-topologies, strong topology on X*, Banach-Alaoglu Theorem, Mackey topology and Mackey space, the topologies of bounded convergence and pointwise convergence.

Completeness in LCS: Barrelled and bornological spaces and their basic properties, equicontinuous family of linear maps between LCS, Banach–Steinhaus theorem, quasi-completeness, precompact sets, open mapping and closed graph theorems.

Recommended Readings

Text book-

- 1. Osborne M.S., Locally Convex Spaces, Springer, 2014.
- 2. Rudin W., Functional Analysis, McGraw Hill, Reprint 2018.

Reference book-

- 1. Narici L. and Beckenstein E., *Topological Vector Spaces*, 2nd edition, CRC Press, 2011.
- 2. Wilansky A., Modern Methods in Topological Vector Spaces, Dover Publication, 1984.

Online resources-

· r · · · · · · · · · ·	
Course Code	: _21MAT
Course Name	: Probability Theory
Credits	: <u>3</u> L- <u>3</u> T- <u>0</u> P- <u>0</u>
Course Type	: Elective
Prerequisites	: Measure and Integration

Course Contents

Algebra and σ -algebra, Borel σ -algebra, Lebesgue Measure, probability measure, basic laws of probability, Measurable functions, random variables, Monotone classes, π -systems, λ systems. Monotone Class Theorem, Distribution function, inverse distribution function, the Fundamental Theorem of Probability, Lebesgue integration, Monotone convergence theorem, Fatou's Lemma, Dominating convergence theorem, Convergence "almost surely (a.s)", convergence in distribution, change of variable formula, expectation of a random variable, Product measure, Fubini's Theorem, independent random variables, Borel-Cantelli Lemmas, Tail σ -algebra, Kolmogorov's 0-1 Law. Various modes of convergence, law of large numbers, Characteristic functions, Moment expansion, characteristics function for Normal distribution. Inversion formula, density formula. Central Limit Theorems.

Recommended Readings

Text book-

1. Billingsley P., Probability and Measure, 3e, Wiley, 1995.

Department/Centre : Department of Mathematics

2. Athreya S. and Sunder V.S., Measure and Probability, Univ. Press, 2008.

Reference book-

- 1. Chung K. L., A course in probability theory, 3e, Academic Press, 2001.
- 2. Bhat B.R., Modern Probability Theory, 2nd edition, Wiley Eastern Ltd., 1985.
- 3. Parthasarathy K.R., Introduction to Probability and Measure, HBA, 2005.
- 4. Bartle R.G., The elements of integration and Lebesgue Measure, Wiley, 1966.

Online resources-

Department/Centre : Department of Mathematics						
Course Code	:	21MAT				
Course Name	:	Flow and	d Thermal Inst	ability		
Credits	:	3	L - <u>3</u>	T - 0	P	
Course Type	:	Elective				
Prerequisites	:	None				

Course Contents

Basic concept of stability/instability - Static and dynamic instability - Temporal stability of inviscid incompressible flows - Squire transformation, Kelvin-Helmholtz stability - Temporal stability of viscous incompressible flows: Orr-Sommerfield problem, channel flow, plane Poiseuille flow, plane Couette flow, Blasius boundary layer flows, unbounded flows - Spatial stability of incompressible flows, incompressible inviscid flows - Centrifugal Instability - Thermal and inertial instabilities: Development of perturbations in space and time, equation of perturbation energy, the Boussinesq equations, free-free boundaries, rigid-rigid boundaries, free rigid boundaries - Convective instabilities: Benard problem, Couette-Benard flow, Rayleigh-Benard convection, Rayleigh-Taylor instability, Marangoni instability, Rayleigh-Benard-Marangoni instability, instabilities in stratified flows, introduction to chaos.

Recommended Readings

Text books-

- 1. Criminale W.O., Jackson T.L., Joslin R.D., *Theory and Computations of Hydrodynamic Stability*, Cambridge University Press, 2018.
- 2. Chandrasekhar S., Hydrodynamic and Hydromagnetic Stability, Dover Publication.

Reference book-

1. Schmid P.J., Henningson D.S., Stability and Transition in Shear Flows, Springer 2001

Department/Centre	: <u>D</u>	epartment of Mathematics
Course Code	:	21MAT
Course Name	:	Fluid Dynamics
Credits	:	<u>3</u> L - <u>3</u> T - <u>0</u> P - <u>0</u>
Course Type	:	Elective
Prerequisites	:	None

Course Contents

Classification of fluids - Eulerian and Lagrangian approach - Irrotational flow - Equipotential surfaces - Mass flux density - Equation of continuity - Conservation of momentum - Integration of Euler's equation under different conditions - Bernoulli's equation - Kelvin's minimum energy and circulation theorems, Potential theorems - Complex potential, Sources, sinks, doublets and vortices, Milne-Thomson circle theorem - Blasius theorem - Three-dimensional flows - Motion of cylinders and spheres - Viscous flow, stress and strain analysis, Stokes hypothesis, Navier–Stokes equations of motion - Steady flow between parallel plates, Poiseuille flow, Steady flow between concentric rotating cylinders.

Recommended Readings

Text book-

1. Chorlton F., Text Book of Fluid Dynamics, CBS Publisher, 2005.

Reference books-

- **1.** Fox R.W., Pritchard P. J. and McDonald A.T., *Introduction to Fluid Mechanics*, Seventh Edition, John Wiley & Sons, 2009.
- 2. Kundu P.K., Cohen I.M., Dowling D.R., *Fluid Mechanics*, Sixth Edition, Academic Press, 2016.

Departmenti	and C Deput ment of municipalities
Course Code Course Name	: 21MAT
	: Measure and Integration
Credits	: <u>3</u> L- <u>3</u> T- <u>0</u> P- <u>0</u>
Course Type	: Elective
Prerequisites	: Real Analysis

Department/Centre · Department of Mathematics

Course Contents

Review of Riemann integrals, Insufficiency of Riemann integrals and the need of more general integrals, Measurable spaces and Measurable sets, Measurable functions and their properties, Measures, Lebesgue and Borel measures, Lebesgue Integration, Fatou's Lemma, Monotone Convergence Theorem, Dominate Convergence Theorem, Comparison of Lebesgue and Riemann Integrals, Lp Spaces, Various Modes of Convergences and their relations, Outer Measures and Generation of Measures, Carathéodary Extension Theorem, Product Measures, Monotone class theorem, Tonelli's and Fubini's Theorem.

Recommended Readings

Text books-

- 1. Bartle R.G., The Elements of Integration and Lebesgue Measures, Wiley Publication, 1966.
- 2. Royden H.L., Real Analysis, 3rd Ed., Prentice Hall of India, 1988.

Reference books-

- 1. Halmos P.R., *Measure Theory*, Springer-Verlag, New York, 1974.
- 2. Kesavan S., Measure and Integration, TRIM, 2019.
- 3. Rudin W., Real and Complex Analysis, 3e, TMH, 1987.
- 4. Stein E.M. and Shakarchi R., Real Analysis- Measure Theory, Integration, and Hilbert Spaces, Princeton University Press, 2005.

Online resources-

1. https://onlinecourses.nptel.ac.in/noc20_ma02/preview

Department/Centre : Department of Mathematics					
Course Code	21MAT				
Course Name	: Advanced Analysis				
Credits	: <u>3</u> L- <u>3</u> T- <u>0</u> P- <u>0</u>				
Course Type	: Elective				
Prerequisites	: Functional Analysis, Measure and Integration				

Course Contents

Review of general positive measure and integrals, Signed Measure, Hahn and Jordan Decomposition Theorems, The Radon-Nikodym Theorem, Complex measures and Radon-Nykodym theorem for complex measures, Total variation norm. L_p spaces, Vitali's convergence theorem, Dual of L_p spaces for $1 \le p < \infty$, linear functionals on L_p -spaces, Chebychev's inequality. Regular Borel measure, Positive linear functionals on $C_c(X)$ for a locally compact Hausdorff space X, Riesz representation theorems, Vitali Carathe'odory theorem, Denseness of $C_c(X)$ in $L_p(X)$ for $1 \le p < \infty$; Riesz-Thorin interpolation theorem.

Recommended Readings

Text books-

- 1. Rudin W., Real and Complex Analysis, 3rd edition, TMH, 1987.
- 2. Aliprantis C.D. and Burkinshaw O., *Principles of Real Analysis*, 3rd edition, Academic Press, 1998.
- 3. Lieb E.H. and Loss M., *Analysis*, 2nd edition, AMS, 2001.

Reference books-

- 1. Kesavan S., Measure and Integration, TRIM, Hindustan Book Agency, 2019.
- 2. Halmos P.R., Measure Theory, Springer-Verlag, New York, 1974.
- 3. Royden H.L., *Real Analysis*, 3rd Ed., Prentice Hall of India, 1988.
- 4. Stein E.M. and Shakarchi R., *Real Analysis- Measure Theory, Integration, and Hilbert Spaces*, Princeton University Press, 2005.

Online resources-

Department/Centre : <u>Mathematics</u>							
Course Code		21MA	21MAT				
Course Name	:	Intro	duction to Ope	erator Theory			
Credits	:	3	L - 3	T - 0	P 0		
Course Type	:	Electi	ve				
Prerequisites	:	Funct	ional Analysis				

Course Contents

Compact Operators and its Properties, Spectral Results for Banach Space operators: Eigen spectrum and spectrum; spectral radius formula; spectral mapping theorem, Riesz-Schauder theory for compact operators. Operators on Hilbert spaces: self-adjoint, normal, unitary operators, Hilbert Schmidt operators. spectrum of various operators on Hilbert spaces, spectral theorem for compact self-adjoint operators; singular value decomposition of compact operators, Spectral theorem for self-adjoint operators, Solution of operator equations.

Recommended Readings

Text books-

- 1. Nair M.T., *Functional Analysis: A First Course*, PHI-Learning, New Delhi, 2002 (Fourth Print: 2014).
- 2. Limaye B.V., Functional Analysis, 2nd Edition, New Age International, 1996.

Reference books-

- 1. Kreyszig E., Introduction to Functional Analysis with Applications, John Wiley & Sons, 1978.
- 2. Nair M.T., *Linear Operator Equations: Approximation and Regularization*, World Scientific, 2009.
- 3. Riesz F. and Nagy B.S., Functional Analysis, Dover Publications, 1990.
- 4. Conway J.B., A Course in Functional Analysis, 2nd Edition, Springer, 1990.

Online resources-

Department/Centre : Department of Mathematics					
Course Code	:	21MAT			
CourseName	:	Geomet	ric Function	n Theory	
Credits	:	3	L - <u>3</u>	T - 0	 P - 0
Course Type	:	Elective			
Prerequisites	:	Complex	x analysis		

Course Contents

Analytic functions, Entire and meromorphic functions, Harmonic functions, Univalent functions.

Maximum modulus theorem, Schwarz lemma, Schwarz function, Weierestrass factorization theorem, Mittag-Leffler theorem, Picards theorems, subordination.

Families of analytic functions: Convex, Starlike, Spirallike, Geometric properties of functions, Conformal mapping on simply connected domains, Mapping properties of special functions, Riemann mapping theorem, Schwarz-Christoffel transformations, Potential function, Laplace equation and solution.

Recommended Readings

Text book-

- 1. Graham I. and Kohr G., *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.
- 2. Krantz S.G., Complex variables, Chapman & Hall/CRC, Indian Reprint 2012.

Reference book-

1. Conway J.B., *Functions of One Complex Variable*, Springer Verlag, NewYork, 2nd Ed., 1978.

- 2. Ahlfors L.V., Complex Analysis, McGraw-Hill, 1966.
- 3. Rudin W., Principles of Mathematical Analysis, McGraw-Hill, 1976.
- 4. Nehari Z., *Conformal Mapping*, Dover publications, NewYork, 1952.
- 5. Lang S., *Complex Analysis*, Springer Verlag, NewYork, 4th ed. 1999.
- 6. Duren, P.L., Univalent Functions, Springer Verlag, NewYork, 1935.
- 7. Hille, E., Analytic Function Theory (Vol. II), 2nd Ed., Chelsea Publications, 1987.

Online resources-

1. Youtube lectures by Bonfert Taylor

Department/Centre : Department of Mathematics				
Course Code	: <u>21MAT</u>			
Course Name	: Fractional Calculus & its Application			
Credits Course Type	: <u>3</u> L - <u>3</u> T - <u>0</u> P - <u>0</u> : Elective			
Prerequisites	Ordinary and partial differential equations, Differential and Integral calculus			

Course Contents

Brief review of Special Functions of the Fractional Calculus, Definition of Mittag-Leffler Functions of one and two parameters, Relations of Mittag-Leffler Function to some other functions, The Laplace transform of Mittag-Leffler Function in two parameters. Wright Function, Definition of Wright function, Integral relation and relation to other functions, Miller Ross function.

The Riemann Liouville Fractional Integral, Fractional Integrals of some functions namely binomial function, exponential, the hyperbolic and trigonometric functions, Bessel's functions, Hyper-geometric function, Dirichlet's Formula, Laplace Transform of the Fractional integral, Leibnitz's Formula for Fractional Integrals. Derivatives of the Fractional Integral and the Fractional Integral of Derivatives.

Derivatives, Properties of Fractional derivatives, Riemann-Liouville fractional derivatives, Riemann-Liouville left-sided derivative, Riemann-Liouville right-sided derivative, Leibnitz's Formula of Fractional Derivatives. Laplace transform of fractional derivatives, Fourier transform of fractional derivatives and Mellin transform of fractional derivatives. Fractional derivatives of standard functions Left and right fractional derivatives.

Definition of Weyl Fractional Integral, Weyl Fractional Derivatives, A Leibniz Formula for Weyl Fractional Integral and simple applications.

Definition Caputo Fractional Derivative, Leibnitz's formula for Caputo fractional derivative, Laplace transform of Caputo fractional derivative, Difference between Caputo fractional derivative and R-L fractional derivative, Caputo left-sided derivative, Caputo right-sided derivative, Caputo-Fabrizio fractional derivative.

(No. of lectures-35)

Recommended Readings

Text Books-

- 1. Miller K.S. and Ross B., *An Introduction to the Fractional Differential Equations*, John Wiley and Sons, 1993.
- 2. Samko S.G., Kilbas A.A., Marichev O.I., *Fractional Integrals and Derivatives*, Gordon and Breach Science Publishers, 1987.
- 3. Kilbas A.A., Srivastava H.M., Trujillo J.J., *Theory and Applications of Fractional Differential Equations*, Elsevier, 2006.

Reference books:-

- 1. Oldham K.B. and Spanier J., The Fractional Calculus, Academic Press Inc., 1974.
- 2. Ricardo A., Dina T., Delfim F.M. Torres, *The Variable-Order Fractional Calculus of Variations*, Springer, 2019.
- 3. Podlubny I., Fractional Differential Equation, Academic Press Inc., 1999.
- 4. Das S., Functional Fractional Calculus, Springer 2011.

Online/E resources:-

Department/Ce e	ntr	: Dep	artment of M	athematics		
Course Code	:	21MAT				
Course Name	:	Calculus of Variations and Tensor Analysis				
Credits	:	3	L - 0	T - 0	P - 0	
Course Type	:	Elective				
Prerequisites	:	Calculus				

Course Contents

Calculus of variations: Basic concepts of calculus of variations, Variation and its properties, Euler's equation, Fundamental lemma of calculus of variation, Functionals dependent on higher order derivatives and on several independent variables, Variational problem in parametric form, applications Variational problem with fixed boundaries, Variational problem with moving boundaries, Sufficient condition for an extremum. Isoparametric problem.

Tensor Analysis: Notation and Definitions of Contravariant and covariant tensors of first and second order. Invariants. Contravariant, covariant and mixed tensors. The Kronecker delta. Algebra of tensors Symmetric and skew-symmetric tensors. Addition and scalar multiplication. Contraction and Quotient law for tensors. Reciprocal Tensor. Definition and properties of first and second kind of Christoffel's symbols, covariant differentiation of tensor, Ricci's theorem, Riemann- Christoffel 's tensor and its properties, Covariant curvature tensor .

Recommended Readings

Text Books:-

- 1. Nayak P.K., *Textbook of Tensor Calculus and Differential Geometry*, PHI Learning, 2012.
- 2. Gupta A.S., Calculus of Variations with applications, PHI Learning, 2005.

Reference Books:-

- 1. Gelfand I.M. and Fomin S.V., Calculus of Variations, Dover Publications, 2000.
- 2. Irgens F., Tensor Analysis, Springer, 2019.

Department/Centre : Department of Mathematics							
Course Code	:	21MAT					
CourseName	: Fractional Differential Equations						
Credits	:	3 L - 3 T - 0 P - 0					
Course Type	-	Elective					
Prerequisites	:	Basic calculus, numerical methods					

Course Contents

Linear Fractional Differential Equations: Fractional Differential Equation of a General Form. Existence and Uniqueness Theorem as a Method of Solution. Dependence of a Solution on Initial Conditions.

The Laplace Transform Method. Standard Fractional Differential Equations. Sequential Fractional Differential Equations.

Fractional Green's Function: Definition and Some Properties. One-Term Equation. Two-Term Equation. Three-Term Equation. Four-Term Equation. General Case: n-term Equation. **Other Methods for the Solution of Fractional-order Equations:** Power Series Method.

Babenko's Symbolic Calculus Method. Method of Orthogonal Polynomials. The Mellin Transform Method.

Numerical Evaluation of Fractional Derivatives: Approximation of Fractional Derivatives. The "Short-Memory" Principle. Calculation of Heat Load Intensity Change in Blast Furnace Walls. Order of Approximation. Computation of Coefficients. Higher-order Approximations.

Numerical Solution of Fractional Differential Equations: Initial Conditions: Which Problem to Solve? Numerical Solution. Examples of Numerical Solutions. The "Short-Memory" Principle in Initial Value Problems for Fractional Differential Equations. Matrix approach to discrete fractional calculus. Numerical solution of nonlinear problems.

Applications. Fractional order systems and controllers.

Recommended Readings

Text book-

- 1. Podlubny. I., Fractional differential equations, Academic Press, 1999.
- 2. Kilbas. A.A., Srivastava, H.M., Trujillo, J.J., *Theory and applications of fractional differential equations*, Elsevier, 2006.

Reference book-

- 1. Miller K.S. and Ross B., An introduction to the fractional calculus and fractional differential equations, John Wiley and SOns, 1993.
- 2. Diethelm K., The analysis of fractional differential equations, Springer, 2004.
- 3. Zhou, Y., Basic theory of fractional differential equations, World Scientific, 2014.

Online resources-

Depai tillenti Cell	111	bepartment of Wathematics				
Course Code	:	21MAT				
Course Name	:	Dynamical Systems				
Credits	:	<u>3</u> L- <u>3</u> T- <u>0</u> P- <u>0</u>				
Course Type	:	Elective				
Prerequisites	:	Ordinary Differential Equations.				

Department/Centre · Department of Mathematics

Course Contents

One-dimensional flows: fixed points and stability, linear stability analysis, Saddle- node bifurcation, Transcritical bifurcation, Pitchfork bifurcation, Flows on the circle. Two-dimensional flows: Linear systems, nonlinear autonomous systems, phase portraits, Fixed points and linearization, conservative systems, index theory, limit cycles, Poincare Bendixson theorem, Lienard systems, Saddle-node, Transcritical and Pitchfork bifurcations, Hopf bifurcation. Chaos. Discrete dynamical systems

Recommended Readings

Text book-

- 1. Strogatz S.H., Nonlinear dynamics and Chaos, Perseus books publishing, 1994.
- Perko L., Differential Equations and Dynamical Systems, 3rd Edition, Springer (India), 2. 2004.

Reference book-

1. Sandefur J.T., Discrete dynamical systems Theory and applications, Clarendon press, 1990.

2. Scheinerman E.R., Invitation to Dynamical Systems. Dover Publications Inc., 2012.

Online resources-

Department/Centre : Department of Mathematics								
Course Code Course Name	:	21MA	21MAT					
	:	Nume	erical Linear A	lgebra				
Credits	:	3	L - <u>3</u>	T - 0	P - 0			
Course Type	:	Electi	ve					
Prerequisites	:	: Linear Algebra and Numerical Analysis.						

Course Contents

Review of basic concepts: Floating-point arithmetic, matrices, operation counts. Matrix multiplication, block matrices. Matrix norms. Linear system sensitivity, conditioning.

Matrix factorizations: Cholesky factorization. QR factorization. LU factorization and Gaussian elimination; partial pivoting. Complete pivoting, rook pivoting. Error analysis. Numerical examples.

Linear least squares problem: Basic theory using singular value decomposition (SVD) and Epseudoinverse. Perturbation theory. Numerical solution: normal equations. SVD and rank deficiency.

Eigenvalue problem: Basic theory, including perturbation results. Power method, inverse iteration. QR algorithm. Arnoldi iteration, Lancoz method.

Iterative methods for linear systems: Review of Jacobi and Gauss-Seidel iterations. Residual corrector method. SOR iterations. Kronecker product. Krylov subspace methods, conjugate gradient method. GMRES. Preconditioning.

Recommended Readings

Text books:

- 1. Demmel J.W., *Applied Numerical Linear Algebra*, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1997.
- 2. Trefethen L.N. and Bau D., *Numerical Linear Algebra*, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA.

Reference books:

- 1. Kincaid D. and Cheney W., *Numerical Analysis: Mathematics of Scientific Computing*, 3rd Ed, Brooks/Cole (2002).
- 2. Datta B.N., *Numerical Linear Algebra and applications*, 2nd Edition, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2010.

Online resources:

Department/Centre : Department of Mathematics					
Course Code	:	21MAT			
Course Name	:	Finite Element Methods			
Credits	:	<u>3</u> L- <u>3</u> T- <u>0</u> P- <u>0</u>			
Course Type	:	Elective			
Prerequisites	:	Basics of numerical methods			

Course Contents

Overview of finite element method, comparison with finite difference methods, Weighted residual methods: Galerkin's, Least squares and Collocation methods. Shape functions, Normalised coordinates.

Variational methods: Functional and its variation, Rayleigh-Ritz method, Equivalence of Rayleigh-Ritz and Galerkin methods in one and two dimensions.

Finite element method: Applications to ordinary differential equations, Elliptic equation, Node-wise assembly, Higher order elements, Element of rectangular shape, Parabolic Equations in one and two dimensions, and Hyperbolic Equations.

Text Books:

- 1. Fish J. and Belytschko T., A First Course in Finite Elements, Wiley 2007.
- 2. Hughes T. J. R., The Finite Element Method, Prentice-Hall, 1986.

Reference Books:

- 1. Cook R. D., Malkus D. S., Plesha M. E. and Witt R. J., *Concepts and Applications of Finite Element Analysis*, 4th Edition, John Wiley, 2002.
- 2. Bathe K. J., Finite Element Procedures, PHI 2006.
- 3. Seshu P., Textbook of Finite Element Analysis, PHI Learning 2018.
- 4. Reddy J. N., *An Introduction to the Finite Element Method*, Third Edition, Tata McGraw-Hill 2014.
- 5. Shrikhande M., *Finite Element Method and Computational Structural Dynamics*, PHI Learning Pvt. Ltd., New Delhi, 2014.

Department/Centre : Department of Mathematics							
Course Code	:	21MAT					
Course Name	:	Integral Equations					
Credits	:	3 L-3 T-0 P-0					
Course Type	:	Elective					
Prerequisites	:	Basic theory of differential equations, Calculus of several variables.					

Course Contents

Definition and classification, conversion of initial and boundary value problems to an integral equation, Eigen-Values and Eigen functions. Solutions of homogeneous and general Fredholm integral equations of second kind with separable kernels.

Solution of Fredholm and Volterra integral equations of second kind by methods of successive substitutions and successive approximations, Resolvent kernel and its results.

Integral equations with symmetric kernels: Complex Hilbert space, Orthogonal system of functions, fundamental properties of eigen values and eigen functions for symmetric kernels, expansion in eigen-functions and bilinear forms, Hilbert-Schmidt theorem.

Solution of Fredholm integral equations of second kind by using Hilbert-Schmidt theorem. Fredholm theorems. Solution of Volterra integral equations with convolution type kernels by Laplace transform.

Text book:

- 1. Wazwaz A. M., *Linear and Nonlinear Integral Equations Methods and Applications*, Springer-Verlag Berlin Heidelberg, 2011.
- 2. Kanwal R. P., Linear Integral Equations, Birkhäuser Basel, 2nd edition, 1997.

Reference Books:

- 1. Mikhlin, S. G., Linear Integral Equations, Routledge, 1961.
- 2. Masujima M., Applied Mathematical Methods of Theoretical Physics Integral Equations and Calculus of Variations, Weinheim, Germany: Wiley-VCH, 2005.
- 3. Lovitte W. V., *Linear Integral Equations*, Dover Publications; Reissue edition, 2005.

Department/Centre : Department of Mathematics							
Course Code	: <u>21MAT</u>						
Course Name	: Number Theory						
Credits	: <u>3</u> L- <u>3</u> T- <u>0</u> P- <u>0</u>						
Course Type	: Open Elective						
Prerequisites	: Knowledge of number system						

Course Contents

Congruences: Basic definitions and properties, complete and reduced residue systems, theorems of Fermat, Euler & Wilson, application to RSA cryptosystem. Linear congruences and Chinese remainder theorem, quadratic congruences, and Quadratic Reciprocity law. **Arithmetical functions**: Euler's Function, τ and σ functions with some properties and their rate of growth, Möbius-Inversion formula.

Diophantine Approximation: Continued fractions and their connection with Diophantine approximations, applications to Pell's equations.

Diophantine Equations: Linear equations, Binary quadratic forms, Solutions of some quadratic and higher degree diophantine equations.

Partitions: Partitions of a number, Some basic properties and results.

Recommended Readings

Text books-

- 1. Burton D., Elementary Number Theory, McGraw Hill, 2006.
- 2. Jones G.A. and Jones J.M., *Elementary Number Theory*, Springer UTM, 2007.

Reference books-

- 1. Baker A., A concise introduction to the theory of numbers, Cambridge Univ. Press, 1984.
- 2. Hardy G.H. and Wright E.M., *An introduction to the theory of numbers*, 4th Editions, Oxford, Univ. Press, 1960.
- 3. Ireland K.F. and Rosen M.I., *A Classical Introduction to Modern Number Theory*, Springer, 1990.
- 4. Niven I., Zuckerman H.S. and Montgomery H.L., An introduction to the theory of numbers, 5e, Wiley, 1991.

Online resources-

1. https://nptel.ac.in/courses/111/103/111103020/

Department/Centre : Department of Mathematics						
Course Code	: 21MAT					
Course Name	: Discrete Mathematics					
Credits	: <u>3</u> L- <u>3</u> T- <u>0</u> P- <u>0</u>					
Course Type	: Open Elective					
Prerequisites	: none; [preferred – understanding of basic mathematics]					

Course Contents

Logic: Propositional Logic, language of propositional logic, truth table, natural deduction, predicate logic: language of predicate logic, Logical inference with Quantifiers. Proof techniques: Introduction to different standard proof techniques.

Set Theory: Review of basic set operations, cardinality of a set. Countable and uncountable sets. Relations, Types of relations, operations of relations and applications, Poset, Congruence arithmetic. Partially ordered sets and Lattices, Hasse Diagrams, lattices as algebraic systems sub-lattices, direct product and Homomorphisms, Complete lattices.

Combinatorics: Counting techniques: Pigeon Hole principle, inclusion exclusion principle, recurrence relation and generating function.

Graphs: Complete graphs, regular graphs, bipartite graphs, Vertex degree, subgraphs, paths and cycles, the matrix representation of graphs, fusion, trees and connectivity, bridges, spanning trees, chromatic number, connector problems, shortest path problems, cut vertices and connectivity.

Recommended Readings

Text books:

- 1. Rosen K.H., *Discrete Mathematics and its Applications with Combinatorics and Graph Theory*, 7th Edition, Tata McGraw-Hill Edu. 2012.
- 2. Liu C.L. and Mohapatra D., *Elements of Discrete Mathematical*, 4th Ed., Tata McGraw-Hill, 2012.

Reference books:

- 1. West D. B., Introduction to Graph Theory, Second Edition, Pearson, 2001.
- 2. Kolman B., Busby R. and Ross S.C., *Discrete mathematical structures*, 4th edition. Prentice Hall of India, 2002.
- 3. Wilson R.J., Introduction to Graph Theory, Fourth Edition, Prentice Hall of India, 1996.
- 4. Bondy J.A. and Murty U.S.R., Graph Theory, Springer, 2008.
- 5. Sane S.S., Combinatorial Techniques, Hindustan Book Agency, 2013.

Online resource-

1. https://nptel.ac.in/courses/106/108/106108227/

Department/Ce	entre	: <u>D</u>	epartment of N	Mathematics				
Course Code	:	21MA	1MAT					
Course Name	:	Classie	cal Mechanics	<u> </u>				
Credits	:	3	L - 3	T - 0	P 0			
Course Type	:	Open H	Elective					
Prerequisites	:	None						

Course Contents

D'Alembert's principle, General equations of motion, Motion of the centre of inertia (Motion of translation), Motion relative to the centre of inertia (Motion of rotation)

Motion about a fixed axis: Moment of effective forces about a fixed axis of rotation, Moment of momentum, Kinetic energy of a rotating body about a fixed line, Principle of angular momentum, Principle of energy and work.

Lagrange's equations of motion: Degrees of freedom and generalized coordinates, Lagrange's equations of motion under finite forces, Lagrangian function, Deduction of principle of conservation of energy from the Lagrange's equations.

Motion in three dimensions: Euler's theorem for motion in three dimensions, system of moving axes, Euler's dynamical equations of motion under finite forces, Kinetic energy of body rotating about a fixed point, Eulerian angles and Euler's geometrical equations, Deduction of Euler's dynamical equations from Lagrange's equations of motion.

Recommended Readings

Text books-

- 1. Rana N.C. and Joag P.S., Classical Mechanics, Tata McGraw-Hill, 2015.
- 2. Goldstein H., Poole C.P. and Safko J., Classical Mechanics, Pearson Education, 2011.

Reference book-

1. Hand L.N. and Finch J.D., Analytical Mechanics, Cambridge University Press, 2008.

Online resources-

Department/Centre : Department of Mathematics							
Course Code	21MAT Introduction to Graph Theory						
Course Name							
Credits	: 3 L - 3 T - 0 P - 0						
Course Type	: Open Elective						
Prerequisites	: UG Mathematics						

Course Contents

Definition and basic concepts, Trees, characterizations, counting of minimum spanning tree, graph and matrices, Paths and distance in graphs, center and median of a graph, activity digraph and critical path, Eulerian graphs, Definition and characterization, Hamiltonian graphs, Necessary and sufficient conditions, Planar Graphs: properties, dual, genus of a graph. Peterson graph.

Graph coloring, vertex coloring, chromatic polynomials, edge coloring, planar graph coloring, Matching and factorizations, maximum matching in bipartite graphs, maximum matching in general graphs, Hall's marriage theorem, factorization, Networks, The Max-flow min-cut theorem, connectivity and edge connectivity, Menger's theorem.

Recommended Readings

Text book-

1. West D. B., Introduction to Graph Theory, Second Edition, Pearson, 2001.

Reference book-

- 1. Bondy J. A. and Murty U. S. R., Graph Theory, Springer, 2008.
- 2. Wilson R. J., Introduction to Graph Theory, Fourth Edition, Prentice Hall, 1996.

Online resources-

1. Graph theory: NPTEL (https://nptel.ac.in/courses/111/106/111106102/)

Department/Centre : Department of Mathematics								
Course Code	:	21MA	21MAT					
Course Name	:	Integ	ral Transform	S				
Credits	:	3	L - <u>3</u>	T - 0	P - 0			
Course Type	:	Open	Elective					
Prerequisites	:	: Ordinary Differential equations, Differential and Integral calculus						

Course Contents

Laplace Transforms : Definition of Laplace Transform and examples, Sufficient conditions for the existence of Laplace Transform, Laplace Transform of some elementary, Properties of Laplace Transform, shifting and scaling properties, Differentiation and Integration properties of Laplace Transform, The Error function, Transform of Bessel function, Periodic functions, Initial & Final Value Theorem, Inverse Laplace Transforms and its examples, Convolution theorem and related examples.

Bilateral Laplace Transform, Relation between Bilateral Laplace transform and Laplace Transform, Some Properties.

Applications of Laplace Transform: Solution of ODE - Initial value problems for linear equations with constant coefficients, Solution of Simultaneous ODE with constant coefficient.

Fourier Transform: Fourier sine and cosine transformation and its examples, Properties of Fourier sine and cosine transform, Transform of Derivatives, Parseval's Theorem, Inversion theorem, Convolution theorem, Relation between Bilateral Laplace transform and Fourier Transform. Applications of Fourier Transform.

Hankel Transform : Elementary properties, Inversion theorem, transform of the derivative of functions, transform of elementary functions, Parseval's theorem relation between Laplace transform and Hankel transform, Basic operational property of Hankel transform

Mellin Transform: Definition and properties of Mellin transform, shifting and scaling properties, Mellin transform of derivative and integral.

Hartley Transform: Definition of Hartley Transform and examples, Inversion theorem, Relation between Stieltjes Transform and Fourier Transform.

Recommended Readings

Text Books:-

- 1. Debnath L., Bhatta D., *Integral transforms and their Applications*, CRC Press/Chapman and Hall, 3rd Edition, 2015.
- 2. Patra B., An Introduction of Integral Transform, Taylor & Francis/CRC, 2018
- 3. Drof R.C., *;Transform and Applications*, Taylor & Francis/CRC, 3rd Edition, 2010

Reference books:-

- 1. Andrews L.C., Shivamoggi B.K., *Integral Transform for Engineers*, SPIE Optical Engineering Press, 1999.
- 2. Sneddon I.N., The use of Integral Transform, McGraw Hill, 2nd Edition, 1972.